

Radians of the Lost Arc

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Grade Level: 10-12

Time Allotment: One 90 min. block

Overview: This lesson will introduce students to the concept and use of radians in angle measurement. Prerequisites: Students should be familiar with π and its relationship to circular objects. Students should be able to measure angles in degrees.

Subject Matter: Geometry, Algebra II, Trigonometry, and Precalculus

Learning Objectives:

The students will be able to estimate angle measurements in both radians and degrees.

Standards:

This lesson addresses the following Virginia SOL's that can be found at:

<http://www.pen.k12.va.us/VDOE/Instruction/sol.html>

- AII/T.22 The student, given the value of one trigonometric function, will find the values of the other trigonometric functions. Properties of the unit circle and definitions of circular functions will be applied.
- AII/T.23 The student will find without the aid of a calculating utility the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This will include converting radians to degrees and vice versa.

Media Components:

- Unitedstreaming: *Math Factor: Radian Measurement: "Defining Radians"*
<http://www.unitedstreaming.com>
- SmartBoard
- Web Sites:
<http://www.mathlearning.net/learningtools/Flash/unitCircle/unitCircle.html>
http://wims.unice.fr/wims/en_H4~geometry~radialen-1.en.html

Materials:

For the Teacher:

- Paper Plates (get the inexpensive white ones that are not coated) Ironed coffee filters may also be used for Learning Activity 3.
- Straight Pins
- Scissors

- Index cards
- Straight edges or rulers to mark lines and flatten plates
- Handouts: Millimeter Beads and Module B: Radians and Degrees (TOPS Learning Systems, Canby, OR 97013) Permission to use this was given by Ron Marson, the author and publisher of the booklet *PI in the SKY*, which contains this activity. For more information go to www.topscience.org.
- Small Post-it-Notes
- Colored Pencils

For each pair of student:

- 3 Paper plates
- Ruler or wooden object to flatten the plate
- Straight pin to find the center of gravity of the plate
- 2 Millimeter Beads Handouts
- Index card
- Scissors
- Colored pencils

Prep for Teachers:

- Prior to teaching, have all Internet sites bookmarked
- You should download and preview the video clip from www.unitedstreaming.com *Math Factor: Radian Measurement: “Defining Radians”*
- Familiarize yourself with the lesson format and be sure to have all materials ready as listed in the materials section.
- When using media, provide students with a **FOCUS FOR MEDIA INTERACTION**, a specific task to complete and/or information to identify during or after viewing of video segments, websites, or other multimedia elements.

Introductory Activity: Setting the Stage

Streaming Video: Unitedstreaming: *Math Factor: Radian Measurement: “Defining Radians”*

1. **Provide a Focus for Media Interaction by saying:** “Today we are going to learn about measuring angles. In the past we have always measured angles in degrees but there is another way. Listen as Nadia and Darryl learn about a different unit of angle measurement. What is this new unit of measurement called?”
2. **Start** the video and **Pause** at 00:26 after you hear Lance say, “Or we can measure an angle using radians.”
3. **Ask:** “What units of measure can we use for angles?” (Degrees and radians)

4. **Focus for Media Interaction:** “Now I want you to be able to describe the task that Nadia and Darryl have been asked to perform.” (Create a central angle that cuts off an arc length of one radius.)
5. **Resume. Pause** at 00:49 when Lance says, “I think you should have enough tools here to do the job.”
6. **Ask:** “What was the task?” (Create a central angle that cuts off an arc length of one radius.) **Replay, if necessary.**
7. **Say:** “Listen for the three steps necessary to create a central angle that cuts off an arc length of one radius.” (First mark an initial side or arm for the angle. Then determine how big the radius is. Mark off this length on the circumference of the dartboard. Then add the terminal side or arm.)
8. **Resume and pause** at 01:37 after you hear, “So this is your angle that cuts off the arc length of one radius. Right? Um hum! OK!”
9. **Ask:** “What were the three steps?” (First mark an initial side (arm) for the angle. Then determine how big the radius is. Mark off this length on the circumference of the dartboard. Then add the terminal side (arm).)
10. **Focus for Media Interaction: Say:** “If you repeated this activity on a different size circle would the central angles be the same size? Watch as Nadia and Darryl repeat the activity using a can of worms.” (Yes, because you have created an angle of one radian.)
11. **Resume and pause** at 2:51 when Lance says, “Are these two angles the same size?” **Say,** “What do you think?”
12. Have students make predictions. (Accept all student responses.)
13. **Resume and pause** at 3:08 when Lance says, “Well, you’re right.”
14. **Focus for Media Interaction:** “Now listen for the formal definition of an angle of one radian and write it in your notebook”
15. **Resume and pause** at 3:22 while the definition is still visible.
16. **Ask:** “What is the formal definition of an angle of one radian?” (An angle of one radian is defined as the angle that is subtended by an arc of measure r , on a circle whose radius is length r .)
17. **Focus:** “Listen for another way to summarize the definition.”
18. **Resume and pause** at 3:36
19. **Ask:** “So what is another way to define a radian?” (The ratio of the length of the arc, which subtends the angle, to the radius. Therefore θ equals arc length over radius when θ is measured in radians.)
20. **Focus for Media Interaction:** “Now that we know that the central angle θ is the arc length over the radius, watch and listen for ways to measure angles such as 2 radians or 5 radians.”
21. **Resume and pause** at 4:59 when you hear Lance say, “Now, do you two feel that you understand a little bit better the idea of what a radian is?”
22. **Ask:** “How did Darryl and Nadia find angles of 2 radians or 5 radians?” (They measured them with the string. Or if you have a radius of 7 cm. and you want to find the arc length when the central angle is 5 radians, you can just multiply 5×7 and get the arc length of 35 cm.)

23. **Focus for Media Interaction:** “You are going to hear one final explanation of an angle of one radian. I’d like for you to be able to tell me an approximate degree measure for an angle of one radian.”
24. **Resume and play** to the end of the segment.
26. **Ask:** “What is the approximate size, in degrees of one radian and how can you remember this?” (57.3°; if you form an equilateral triangle with two of the sides on the radii of the circle it is just a tad bigger than the angle of one radian.)

Learning Activity, Part 1: So What’s a Radian?

Have students replicate the activity in the video as described below or in TOPS PI in the Sky, activity B1: *So what’s a radian* page 26 (attached)

1. For each pair of students, hand out a pin, pair of scissors, paper plate and straight edge.
2. Have students balance their paper plate on the pinhead and then once it is balanced, punch the pin through the plate to mark the center.
3. Flatten all plates with a ruler or other flat object.
4. Place the corner of an index card on the center of the paper plate. Using the long side of the index card as a straight edge, mark the radius of the circle on the plate, and then mark the length of the radius on the index card. Cut the card the length of the radius.
5. Cut the plate (or just mark it) into radius wedges whose sides are all equal. You will have two straight sides and one curved side. Radian wedges are created by bending the index card around the curve of the plate and marking that distance as the third side. Label each side of the wedge with the words “1 radius”. **Ask:** “How many radius wedges did you get? Did you get any partial wedges?” (Students should get six complete radian wedges and a fractional wedge.)
6. Tell students that each radius wedge represents the central angle of one radian. Therefore a radian is the angle that is subtended (cut off by) by the arc that is the same length as the radius of the circle.
7. Have students notice that the fractional wedge should appear to be about a fourth of the radian wedge.
8. Explain that since the entire circle is not an integral multiple of radians, familiar angles are expressed in terms of π radians.
9. **Say:** “Let’s do the math! Since the formula for the circumference of a circle is $C=2\pi r$, and there are six and a little more radian wedges which means there are six and a little more times the radius, and it just so happens that 2π is 6.28 therefore, to measure familiar angles, we use π and fractions of π . For example, if we want to measure a semicircle, that would be π radians or 180° . From this fact we can convert 90° to $\pi/2$ radians, 45° to $\pi/4$ radians and so on.”

Learning Activity, Part 2: Millimeter Beads Activity

Distribute Millimeter Beads Handout and colored pencils.

Say: “Let’s confirm our hypothesis from the previous activity. Does the radius actually fit 2π or 6.28 times around the circle? If that is the case, then there are 2π radians in a circle. Let’s check to see if that ‘little bit more’ is actually .28 more.”

1. Have students use circle Y on the Millimeter Beads sheet. (Attached)
2. Count the number of beads in the radius. (70)
3. Mark off one radian angle. (70 beads around the circle)
4. Color the first set of beads.
5. Mark off the next radian angle. (70 more beads)
6. Color that length of beads a different color.
7. Repeat until you have marked and colored as many lengths as possible. (6 full lengths)
8. Count the remaining beads.
9. There should be 20 remaining beads. Divide that number by the radius (20/70) and what do you notice when you add the six whole sections to that fraction? (6.28571...) whereas $2\pi = 6.2831$
10. Explain to the students that multiples of π are convenient when we measure common angles in radians.

Learning Activity, Part 3:

1. Each student will need a flattened paper plate (or ironed coffee filter) and a straight edge.
2. Have each student fold a plate in half and then in quarters.
3. Mark the angles as 0° , 90° , 180° , 270° and 360° . Then mark the angles in radians, 0π , $\pi/2$, π , $3\pi/2$ and 2π .
4. Next have students fold each of those angles in half to create 45° , 135° , 225° , and 315° and mark the matching radian measures of $\pi/4$, $3\pi/4$, $5\pi/4$, and $7\pi/4$.
5. Next, have the students fold their plates in half, then take that half and divide it in thirds by folding $1/3$ of the plate over the rest of it making a funnel before flattening the plate into thirds.
6. Mark those lines as 60° , 120° , 240° and 300° .
7. Mark the corresponding radian measures of $\pi/3$, $2\pi/3$, $4\pi/3$, and $5\pi/3$.
8. Finally fold those angles in half to make 30° angles.
9. Mark them 30° , 150° , 210° , and 330° and mark the radians as $\pi/6$, $5\pi/6$, $7\pi/6$, and $11\pi/6$.

Remind students that negative measurements go in a clockwise direction. These plates will be used as a study guide.

Culminating Activity:

- Write all radian and degree angles from Learning Activity 3 on individual post-it-notes.

- Go to <http://www.mathlearning.net/learningtools/Flash/unitCircle/unitCircle.html> and project the Interactive Unit Circle on a Smart Board.
- Hand post-it-notes to individual students and have them place the corner of the post-it-note on the interactive unit circle to mark the angle location. To check for accuracy, move the point along the unit circle until the correct angle is reached. Be sure and practice both degree and radian locations.

Assessment:

If you have access to individual computers for student use, have students go to either of these sites to take a quiz to determine mastery.

<http://www.mathlearning.net/learningtools/Flash/unitCircle/unitCircle.html>

http://wims.unice.fr/wims/en_H4~geometry~radialen-1.en.html

Community Connections:

Invite an electronic engineer to class to describe the use of radians in expressing frequency and phase angles when analyzing or designing electronic circuits.

Cross Curricular Extensions:

- Art: Have students decorate their plates.
- Music: Students can make up songs to remember how to convert radians to degrees and degrees to radians.
- Science: Refer to Pi in the SKY activity booklet for additional activities involving radians.