

“X” Marks the Spot

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Overview: In this lesson, the graph of a pair of linear equations represents the movement of two runners. The intersection of the runners’ paths is the intersection of the two linear equations. These runners have the option of changing direction, speed, and beginning location and seeing how this affects the point of intersection. Students physically experience this by using a graphing calculator, CBL, and motion detector to record their movement and calculate the point of intersection. The exploration is extended online where students are able to manipulate the walking speed, direction, and beginning location of two people and find the intersection. Finally students use their understanding of slope, y-intercept, and equations to write and solve the system of linear equations graphically.

Grade Levels: Algebra I/II

Time Allotment: Three 50-minute class periods

Subject Matter: Mathematics

Learning Objectives:

Students will be able to:

- Collect data using the graphing calculator, CBL, and motion detector.
- Predict the intersection of two lines.
- Use speed, direction, and beginning location to write a linear equation.
- Solve a system of linear equations graphically.

Standards:

This lesson addresses VA. SOL Mathematics A.8 and A.9 available at www.pen.k12.va.us

Media Components:

Video: *Using a System of Equations to Predict Where Two Runners Meet*

This video is located at the web site: <http://www.unitedstreaming.com> . Here you will enter your school’s password and username. Under search, type in the title of the video and this will take you directly to its location.

Web site: <http://standards.nctm.org/document/eexamples/chap5/5.2/index.htm>

Materials:

Materials needed for the Introductory Activity and Culminating Activity:

- Computer with access to the Internet and large screen TV or projection device
- Overhead projector, blank overhead transparencies, transparency pens
- Transparency of a coordinate plane

- TI-83 overhead graphing calculator and view screen
- TI-83 calculator
- 2 unit-to-unit link cables for the graphing calculators
- 2 motion detectors (Available from <http://www.vernier.com>)
- 2 Calculator Based Laboratories (CBL) (Available from <http://www.vernier.com>)
- MEETYOU program for the graphing calculator loaded onto both TI-83 calculators. (The program can be downloaded from the disk that comes with Real World Math with the CBL System Brueningsen, Bower, Antinone, Brueningsen, Texas Instruments)
- Stopwatch
- 3 meter sticks
- Answer Key for Exploration Worksheets 1 and 2 (attached)

For each pair of students:

- Computer with access to the Internet
- Handouts-Exploration Worksheets 1 and 2

Prep for Teachers

Prior to teaching this lesson, bookmark the sites listed above. Visit the web sites and work through the student Exploration Worksheets 1 and 2.

Make a transparency and student copies of the student handouts.

Download the streaming video and practice using the play, pause, and volume features.

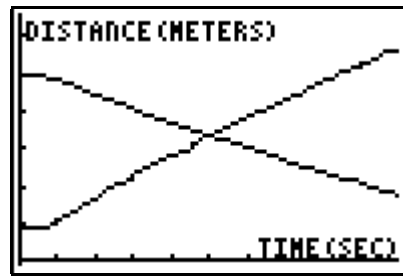
Cue the video to the beginning of the viewing segment. Familiarize yourself with the audio and visual cues used in the Learning Activity portion of the lesson. Load the program “MEETYOU” into the graphing calculators. Do the Introductory Activity.

Introductory Activity: Setting the Stage

1. Say, “ Today we will be investigating the importance of looking at graphs of data. One thing that statisticians and scientists are always interested in is whether or not there is a linear pattern in the data. Whenever a linear pattern can be identified, we can use the graph to make predictions and decisions. In our first activity, we will be using the motion detector to collect data representing the motion of two walkers walking towards each other. The calculator will graph the data. Then, we will use the graph to determine the point of intersection, in other words, when and where the walkers’ paths crossed.
2. Clear a walkway about four meters long and two meters wide. Place a table at one end of the walkway. Place the motion detectors on the table and about one meter apart from each other. Down the middle of the walkway, beginning one-half of a meter from the edge of the table, place the three meter sticks end to end. (A diagram of the setup is provided at the end of the lesson.)
3. Say, “ First I need two volunteers to walk. The first walker will start three meters in front of motion detector #1 and will walk toward the motion detector. Walker number two will start one-half of a meter in front of motion detector #2 and walk away from the motion detector.” Position the volunteers.

4. Say, “ Before we continue with the setup, let’s decide what our variables are going to be. Remember we have two people walking towards each other and we are finding the time and location of their intersection.”
5. Ask, “What two units will we be measuring?” (time and distance)
6. Say, “ Since we will be measuring time and distance these are our two variables.”
7. Say, “ Now ask yourself which makes more sense, the distance traveled depends on the amount of time or the time depends on the distance?” (distance depends on time)
“Since the distance traveled depends on the time, distance is the dependent variable or resulting variable and time is the independent variable or the control variable.”
8. Place the transparency of the coordinate plane on the overhead projector. Label the x-axis “time” and the y-axis “distance”. Say, “We are labeling the x-axis “time” since it is the independent variable. We are labeling the y-axis “distance” since it is the dependent variable and the distance the walker walks depends on the amount of the time that has elapsed.” Remove the transparency and save it for a later discussion.
9. Say, “Next, we need someone to operate the stopwatch. The timer will start the stopwatch when the walkers begin and stop it when their paths intersect.”
10. Say, “ We will also need two people to operate the calculators.” Position these students beside the calculators.
11. Say, “ Finally, we need someone to mark the spot on the meter sticks where the two walkers paths intersect.” Position the intersection marker volunteer.
12. Say, “ Now let’s begin the experiment.” Have calculator operators turn on the graphing calculators. Instruct them to press the PRGM key, arrow down to the MEETYOU program, press ENTER to select the program and ENTER to run the program. Choose 1: COLLECT DATA by pressing ENTER. Follow the directions on each of the graphing calculator screens. These directions consist of using the unit-to-unit link cable to connect the CBL to the graphing calculator, hooking up the motion detector to the sonic port, and turning on the CBL. Each calculator will check to make sure all of the connections are correct and state that the status is O.K. The remaining directions will appear on the calculator screen depending on the designation of the walker as WALKER 1 or WALKER 2.
 - a. SELECT ONE: (Choose WALKER 1 for the overhead calculator and WALKER 2 for the other graphing calculator.)
 - b. Directions for WALKER 1 are as follows:
 1. Stand about 3 meters from the motion detector to start.
 2. When the CBL is activated, move towards the detector at a slow and steady rate.
 3. Be sure you do not come closer than $\frac{1}{2}$ meter from the motion detector while the CBL is sampling.
 4. You will have about 10 seconds to complete the motion. Be sure the stopwatch timer is ready.
 5. Hit [ENTER] on both calculators simultaneously to begin.
 6. Link calculators and hit [ENTER].
 - c. Directions for WALKER 2 are as follows:
 1. Stand about $\frac{1}{2}$ meter from the motion detector to start.
 2. When the CBL is activated, move away from the detector at a slow and steady rate.

3. Be sure you do not come (start) closer than $\frac{1}{2}$ meter from the motion detector while the CBL is sampling.
 4. You will have about 10 seconds to complete the motion. Be sure the stopwatch timer is ready.
 5. Hit [ENTER] on both calculators simultaneously to begin.
 6. Link calculators now. DONE
13. The timer is to begin the stopwatch when the sampling begins and stop it when the two walkers meet each other. The intersection marker is to mark the spot on the meter stick where the intersection occurs.
 14. The graph should appear on the screen of the overhead calculator and look like the graph below. If the graph is not similar to the one shown, redo the experiment.



15. Once a good graph is obtained ask, “What does this graph mean and why is there an X on the screen?” (It shows the graphs of the lines of the two walkers and their point of intersection.)
16. Ask, “What were the units of time?” (seconds) Ask, “What were the units of distance?” (meters)
17. Say, “Can anyone estimate from the graph the time and location of the intersection?” (accept all answers) Say, “Let’s look at the graph by tracing until we get to point where the two lines meet.” Use the trace and left and right arrow keys on the graphing calculator to move the cursor until it is at the point of intersection. Write the ordered pair on the board and ask, “What does this ordered pair mean?” (the x-coordinate is the amount of time taken and the y-coordinate is the distance from the motion detector) Say, “In other words it took _____ (fill in the x-coordinate value) seconds and the walkers met _____ meters (fill in the y-coordinate value) from the motion detectors. Let’s compare our findings with our timer and our intersection marker.” Have the timer and intersection marker report the time and distance. This should compare favorably with the x- and y-coordinates.
18. Summarize by saying, “We can see that a graph can be used to show motion. Then using the graph, we can find the intersection.”

Learning Activities

1. Provide the students with a **Focus for Media Interaction**, and say, “Now that we have seen how a graph can be used to find the intersection of two walkers let’s see what goes into determining where the walkers meet.”
2. Go to the web site

<http://standards.nctm.org/document/eexamples/chap5/5.2/index.htm>

3. Say, "This web site provides us with a simulation of two runners. Shortly you will be working in pairs and will be exploring various rates and starting distances for the two runners. Then you will be making predictions as to where the runners will meet. Let's try a few simulations together."
 - a. Say, "First we use the stop button to reset the simulation. Now, by clicking on their icons, we will set the two runners facing each other with the girl's position at 0 and the boy's position at 100. What does this mean?" (the runners are 100 units away from each other)
 - b. Say, "Now by using the down arrow, let's set the girl's pace at 1 and the boy's pace at 1. This represents one step per unit of time. For convenience, let's let each unit of time represent 1 second, and each step unit represent 1 foot."
 - c. Ask, "When do you think the two runners will meet?" (accept all answers)
 - d. Say, "Let's run the simulation by pressing play."
 - e. Ask, "Where did the runners meet? (50, 50)"
 - f. Ask, "What does this point represent?" (they traveled for 50 seconds and met 50 feet from the beginning point)
4. Say, "Let's try another simulation. Notice we press stop to reset the simulation."
 - a. Say, "Now by dragging the runner and using the up arrow, let's set the girl at 0 with a step of 2 and set the boy at 100 with a step of 2. We will have the runners facing each other. We press stop to reset the simulation."
 - b. Ask, "When and where do you think the runners will meet?" (accept all answers)
 - c. Say, "Let's run the simulation and find out."
 - d. Ask, "Where did they intersect?" (25, 50)
 - e. Ask, "What does this mean and why do you think the intersection was at the same distance but half of the time?" (they traveled for 25 seconds and met 50 feet from the beginning point because they were each running twice as fast so it took $\frac{1}{2}$ of the time)
5. Say, "For our final simulation, let's set the girl at 0 with a step of 3 and the boy at 100 with a step of 3. Have the runners facing each other. Press the stop to reset the simulation."
 - a. Ask, "When and where do you think the runners will meet?" (accept all answers)
 - b. Say, "Let's run the simulation and find out."
 - c. Ask, "Where did they intersect?" ($16\frac{2}{3}$, 50)
 - d. Ask, "What does this mean and why do you think the intersection was at the same distance but $\frac{1}{3}$ of the time?" (they met at 50 feet because they traveled a total of 100 feet and were running at the same pace so each runner would cover $\frac{1}{2}$ of the distance) They were running 3 times as fast so they covered the distance in $\frac{1}{3}$ of the time.
6. Say, "Now it's your turn. You and your partner will be doing two Explorations worksheets. While these are being distributed please go to the following web site:"

<http://standards.nctm.org/document/eexamples/chap5/5.2/index.htm>.

Once worksheets are distributed and students have located the web site say, "Remember you will set the starting position for each runner by dragging the icons along the tracks."

By clicking once on their icons, you will be able to change the direction in which they are running. The pace or length of the stride for each runner can be changed by using the controls on the lower left.”

Say, “Once the runners’ beginning positions, directions, and length of stride have been set, it will be your job to predict when and where the runners will meet. For discussion purposes although the simulation runs much faster, we will call the units of time “seconds” and the units of distance “feet”.”

Make sure all students understand how to use the interactive features of the web site.

7. Once students have finished the Exploration Worksheets, discuss and correct their findings. (An answer key is attached.) Have students discuss which Explorations were relatively easy and which were more difficult. Students should recognize when the runners were running at the same pace they met at the halfway point and the time was related to the length of the stride. A stride of 1 took 50 seconds, a stride of 2 took 25 seconds, and a stride of 4 took $12\frac{1}{2}$ seconds. Students should recognize that when the runners were running at two different paces the ability to predict the intersection became much more difficult.

8. Provide students with a **Focus for Media Interaction** and say, “Wouldn’t it be nice to have a faster way other than guessing and checking to find out the intersection of the two runners.” Say, “We’re going to watch a video clip about two runners and what they did to find out their point of intersection. Listen carefully to find out the setting for the race and the predicament the two runners are facing.” Begin the streaming video at the beginning of the clip, “*Using a System of Equations to Predict Where Two Runners Meet*”.

- a. **Pause** at 02:50 when Pam says, “I’ll figure it out and give you a call.” Ask, “Where are the two girls and what’s their problem?” (The two sisters have gotten separated in the Bay to Breakers foot race in San Francisco)
- b. **Refocus** students by saying, “Listen carefully for what Pam proposes to do in order to help her figure out where she and Betsy can catch up with each other.”
- c. **Resume** and **Pause** at 03:10 when Pam says, “We can make a pretty good graph showing how Betsy will run the race and how I will run the race.”
- d. Ask, “What was Pam’s proposal?” (to make a graph of how she and Betsy will run the race)
- e. **Refocus** by asking, “How will Pam use the graphs to show when she and Betsy will be at the same place?”
- f. **Resume** and **Pause** at 04:28 when Pam says, “At the point where the two lines intersect, Betsy and I will be in the same place at the same time.”
- g. Ask, “How is Pam going to use the graphs?” (the graph will show the point of intersection which will tell at what time the two girls are in the same place)
- h. **Refocus** students by saying, “Listen for what this point of intersection tells the girls.” **Resume** and **Pause** at 4:35 when Betsy says, “Forty minutes after I start running we should both be at the four mile marker.”

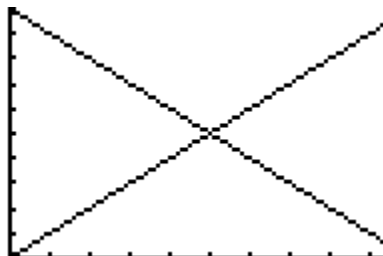
- i. Ask, "What did this intersection tell the girls?" (the intersection tells that the girls will meet 40 minutes after Betsy starts running the race and at the 4 mile marker)
- j. **Refocus** students by asking, "So what is the benefit of being able to develop equations and graphs for how the girls will run the race?"
- k. **Resume** and **Pause** at 05:41 when Pam says, "And it allows us to figure out when and where we will meet along the racecourse."
- l. Say, "Can you summarize and tell me why is it helpful to be able to develop the equations and graphs?" (if we can get equations for the race then we can graph the two equations and find the point of intersection)

Culminating Activity

1. Say, "Let's see if Pam's and Betsy's strategy of writing equations can help us determine when and where our two runners will meet.
 - a. Post the transparency showing the x and y axes labeled with "distance" and "time". Ask, "What are the two values needed to write an equation in $y = mx + b$ form?" (m, the slope and b, the y-intercept)
 - b. Say, "Let's look at our first situation where we set the two runners facing each other with the girl's position at 0 and the boy's position at 100 and their paces at 1."
 - c. Say, "Slope is the change in distance over the change in time, in other words the number of feet per second. Since the step represents how far the runner moves each second, this step is the slope of the equation."
 - d. Say, "Since the distance from 0 is increasing for the girl, we know that her slope is +1. Since b represents the y-intercept or the distance at which the walker begins, we know that b for the girl's equations equals 0." Ask, "So if $m=+1$ and $b = 0$, what is the equation for the girl?" ($y = 1x + 0$ or $y = x$.)
 - e. Say, "Now let's look at the boy's walking. Again, we will use the formula $y = mx + b$. What is m or the number of feet per "second"; and is this distance from 0 increasing or decreasing?" (1 and decreasing) Say, "Since the boy's distance is decreasing m will equal -1 ."
 - f. Say, "Now we need to find where the boy begins. Where does he begin?" (at 100).
 - g. Ask, "Can you tell me what this equations will be?" ($y = -1x + 100$)
 - h. Say, "Now that we have our two equations, let's enter them in the graphing calculator."
 - i. Say, "Let's make sure all of our calculators have the same display."
 - j. Ask students to follow along with their calculators as you model on the overhead calculator.
 1. Press MODE and check to see if everything is left justified.
 2. Press WINDOW and set your window as:
 - Xmin: 0
 - Xmax: 94
 - Xscl: 10
 - Ymin: 0
 - Y max: 100

Yscl: 10

3. Press 2nd Format and check to see if everything is left justified.
4. Press 2nd, STATPLOT and select 4: PlotsOff. Press ENTER, ENTER.
5. Press Y=. In y_1 type $1x + 0$ or simply $1x$, in y_2 type $-1x + 100$
6. Now press graph and the graph should appear. See sample below.



- k. Ask, “What do you think the X on the screen means?” (it is where the two walker’s path’s cross)
- l. Say, “Let’s use the TRACE key along with the left and right arrows to locate this point. Once you are at the point press the up and down arrow keys to verify that the point is common to both lines.”
- m. Ask, “What was the point and what does it mean?” (the point (50,50) means that the two runners will meet after 50 seconds and 50 feet from 0)
- n. Continue to rework the examples in the sections 3 and 4 of the Learning Activities and verify that the intersection of the graphs of the equations does provide the solution as to when and where the two runners will meet. For the solution $(16\frac{2}{3}, 50)$ students will need to use the ZOOM feature of the graphing calculator or press 2nd Calc, Intersection, ENTER, ENTER, ENTER.

2. Have students return to Exploration 1 Worksheet, create and record the two equations for each situation, graph the graphs, and find the intersection. Once students have completed Exploration 1 have them discuss the power and effectiveness of using equations and graphs. Continue the same procedure with Exploration 2.

Cross-Curricular Extensions

Biology: Have students investigate the use of graphs and mathematical models to predict and compare the growth of disease or populations. The following web site contains information about the number of cases for a variety of diseases, <http://www.cdc.gov/ncidod/dbmd/diseaseinfo/> . Under search enter the word *data*.

Environmental Science: Have students collect data and use graphs to investigate the progress made in protecting certain endangered species such as the California condor or the Florida manatee. A great source for environmental data is http://www.seattlecentral.org/qelp/Data_MathTopics.html .

Mathematics: Have students go to the web site, <http://www.exploremath.com/activities/> and select Lines/Linear Equations. Next, select 2 X 2 linear equations. Here students will

define a 2 X 2 linear system and view the corresponding lines and their intersection. They may also view the equations in slope-intercept form and examine the value of the determinant.

Community Connections





- Have a pediatrician discuss with students the use of growth charts in the medical field.
- Have a stockbroker come and discuss the use of graphs in sharing information with clients and making decisions.
- Have a banker illustrate the growth patterns of accounts with different interest rates, compounded rates, different amounts of principal, and the affect of time.

Student Materials

Exploration Worksheets 1 and 2

The goal of this activity is to have you, the learner, determine when the two runners' paths will intersect. You will need to specify the elapsed time and number of feet from the house at which this intersection occurs.

EXPLORATION 1: In this scenario the two runners will be running towards each other, with the same stride yet from a variety of starting positions. Your job is to predict the time and location of their intersections. For discussion purposes, each unit of time will be referred to as one second and each unit of distance one foot.

Situation 1: By dragging the icons, position the girl runner at 0 and the boy runner at 100. Click on the runners' icons so that they are facing each other. Use the down arrows to set the stride of each runner at 1. Predict when and where the two runners will intersect. Record your prediction. Now press the square icon to activate the simulation. If you would like the simulation to run in slow motion, one second per click, use the  icon; otherwise, run the simulation using the  icon. In order to pause the simulation, click on the  icon. To reset the simulation, press the  icon. Record the actual intersection.

Prediction: _____ Actual: _____

Situation 2: Position the girl at 0 and the boy at 50. Keep all other settings the same. Make your predictions. Now press the square icon to activate the simulation. Run the simulation using the motion of your choice. Record the actual intersection.

Prediction: _____ Actual: _____

Situation 3: Position the girl at 10 and the boy at 50. Keep all other settings the same. Make your predictions. Now press the square icon to activate the simulation. Run the simulation. Record the actual intersection.

Prediction: _____ Actual: _____

Situation 4: Position the girl at 20 and the boy at 80. Keep all other settings the same. Make your predictions. Now press the square icon to activate the simulation. Run the simulation. Record the actual intersection.

Prediction: _____ Actual: _____

Situation 5: Position the girl at 0 and the boy at 100. Using the up arrow set each runner's stride at 2. Make your predictions. Now press the square icon to activate the simulation. Run the simulation. Record the actual intersection.




Prediction: _____ Actual: _____

Situation 6: Position the girl at 0 and the boy at 100. Using the up arrow set each runner's stride at 4. Make your predictions. Now press the square icon to activate the simulation. Run the simulation. Record the actual intersection.

Prediction: _____ Actual: _____

Summarize your findings for EXPLORATION 1.

EXPLORATION 2: In this scenario the two runners will be running towards each other with varying strides. Again, your job is to predict the time and location of the intersection.

Situation 1: Position the girl at 0 and the boy at 100. Set the girl's stride at 2 and the boy's stride at 1. Have the runners facing each other. Make your predictions. If you would like the simulation to run in slow motion, one second per click, use the  icon; otherwise, run the simulation using the  icon. In order to pause the simulation, click on the  icon. Now press the square icon to activate the simulation. Run the simulation and record the intersection.

Prediction: _____ Actual: _____

Situation 2: Position the girl at 0 and the boy at 100. Set the girl's stride at 1 and the boy's stride at 2. Have the runners facing each other. Make your predictions. Now press the square icon to activate the simulation. Run the simulation and record the intersection.

Prediction: _____ Actual: _____

Situation 3: Position the girl at 0 and the boy at 100. Set the girl's stride at 3 and the boy's stride at 1. Have the runners facing each other. Make your predictions. Now press the square icon to activate the simulation. Run the simulation and record the intersection.

Prediction: _____ Actual: _____

Situation 4: Position the girl at 0 and the boy at 100. Set the girl's stride at 1 and the boy's stride at 3. Have the runners facing each other. Make your predictions. Now press the square icon to activate the simulation. Run the simulation and record the intersection.

Prediction: _____ Actual: _____

Situation 5: Position the girl at 0 and the boy at 100. Set the girl's stride at 3 and the boy's stride at 2. Have the runners facing each other. Make your predictions. Now press the square icon to activate the simulation. Run the simulation and record the intersection.

Prediction: _____ Actual: _____

Situation 6: Position the girl at 0 and the boy at 100. Set the girl's stride at 2 and the boy's stride at 3. Have the runners facing each other. Make your predictions. Now press the square icon to activate the simulation. Run the simulation and record the intersection.

Prediction: _____ Actual: _____

Summarize your findings for EXPLORATION 2. How was the accuracy of your predictions affected by having differing step values for the boy and girl?

ANSWER KEY FOR STUDENT EXPLORATIONS

EXPLORATION 1: (time in seconds, distance in feet from the house)

1. (50,50)
2. (25,25)
3. (20,30)
4. (30,50)
5. (25, 50)
6. (12.5, 50)

Sample Summary: Since the runners are running at the same speed but in opposite directions, they will each travel exactly half of the distance between them and will meet at the point that is halfway between their starting positions. The time it takes the runners can be found by subtraction the two beginning positions to find the number of feet that need to be run and dividing this value by the sum of the two runners paces. This distributes the amount of distance evenly between the two runners.

EXPLORATION 2:

1. ($33\frac{1}{3}$, $66\frac{2}{3}$)
2. ($33\frac{1}{3}$, $33\frac{1}{3}$)
3. (25, 75)
4. (25, 25)
5. (20, 60)
6. (20, 40)

Sample Summary: The time it takes the runners can be found by subtracting the two beginning positions to find the number of feet that need to be run and dividing this value by the sum of the two runners paces. This distributes the amount of distance proportionally between the two runners. Finding the point at which the runners meet is more difficult since the runners are not running at the same speed. This point can be found by focusing on the girl. Since she is always beginning at 0 first figure out what fraction of the race she ran by taking her pace divided by the sum of the paces. Then multiply this value by the distance between the runners and this will give you the number of feet she traveled. Now add this value to her starting position, 0, and you will have the distance at which the two runners meet. Using the boy's information will also work, just subtract the number of feet he ran from his beginning position, 100.